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What are the bounds on space-time non-commutativity?

X. Calmet[a]

California Institute of Technology, Pasadena, California 91125, USA

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Abstract. In this article we consider the bounds on the non-commutative nature of space-time. We argue that these bounds are extremely model dependent. In the only phenomenologically viable framework, i.e. when the fields are taken to be in the enveloping algebra, the constraints are fairly loose and only of the order of a few TeV. We concentrate on the most stringent bounds that come from clock comparison experiments. In the framework where fields are taken in the enveloping algebra, they are model independent since these bounds are independent on choices involved with the Seiberg–Witten maps.

The aim of this work is to discuss the bounds on the non-commutative nature of space-time. We will argue that these bounds are extremely model dependent and in particular depend largely on whether the non-commutative fields are Lie algebra valued or in the enveloping algebra. For reasons that will be explained later, the only phenomenological viable approach is the one where fields are assumed to be in the enveloping algebra. It turns out that in that case the bounds are fairly loose and are of the order of a few TeV only.

The idea that space-time might be non-commutative at short distances is not new and can be traced back to Heisenberg [1], Pauli [2] and Snyder [3]. This idea was taken very seriously recently because non-commutative coordinates were found in a specific limit of string theory. This is nevertheless not the only motivation to study Yang–Mills theories on non-commutative spaces. In the early days of quantum field theories, it was thought that a fundamental cutoff might be useful to regularize the infinities appearing in these theories. Nowadays it is understood that gauge theories describing the strong and electroweak interactions are renormalizable and thus infinities cancel, but it might still be useful to have a fundamental cutoff to make sense of a quantum theory of gravity, whatever this might be. A more pragmatic approach is that space-time could simply be non-commutative at short distances in which case one has to understand how the standard model can emerge as a low energy model of a Yang–Mills theory formulated on a non-commutative space-time.

The simplest non-commutative relations one can study are

$$[\hat{x}^\mu, \hat{x}^\nu] \equiv \hat{x}^\mu \hat{x}^\nu - \hat{x}^\nu \hat{x}^\mu = \mathrm{i}\theta^{\mu\nu}, \qquad \theta^{\mu\nu} \in \mathbb{C}. \tag{1}$$

Postulating such relations implies that Lorentz covariance is explicitly broken. These relations also imply uncertainty relations for space-time coordinates:

$$\Delta x^\mu \Delta x^\nu \geq \frac{1}{2}|\theta^{\mu\nu}|, \tag{2}$$

which are a reminiscence of the famous Heisenberg uncertainty relations for momentum and space coordinates. Note that $\theta^{\mu\nu}$ is a dimension-full quantity, $\dim(\theta^{\mu\nu}) = \text{mass}^{-2}$. If this mass scale is large enough, $\theta^{\mu\nu}$ can be used as an expansion parameter like $\hbar$ in quantum mechanics. We adopt the usual convention: a variable or function with a hat is a non-commutative one. It should be noted that the relations (1) are very specific and other relations have been considered. Other examples are Lie structures, $[\hat{x}^\mu, \hat{x}^\nu] = \mathrm{i}f^{\mu\nu}_\alpha \hat{x}^\alpha$, and quantum plane structures, $[\hat{x}^\mu, \hat{x}^\nu] = \mathrm{i}C^{\mu\nu}_{\alpha\beta}\hat{x}^\alpha \hat{x}^\beta$. It is known how to formulate Yang–Mills theories on a generic Poisson structure [4].

The aim of this work is to discuss the bounds on space-time non-commutativity appearing in the literature. It should be noted that most bounds on the non-commutative nature of space-time come from constraints on Lorentz invariance. These constraints are extremely model dependent. There are different approaches to gauge field theory on non-commutative spaces. The first approach is motivated by string theory, see e.g [5] for a review. It is non-perturbative in θ and the non-local property of the interactions is manifest. Fields are taken as usual to be Lie algebra valued. Unfortunately it turns out that this approach suffers from a number of drawbacks that make it unsuitable to build realistic models for the electroweak and strong interactions.

If fields are assumed to be Lie algebra valued, it turns out that only U(N) structure groups are conceivable (see [5] for a review). This approach cannot be used to describe particle physics since we know that SU(N) groups are required to describe the weak and strong interactions. Or at least there is no obvious way known to date to derive the standard model as a low energy effective action coming

[a] e-mail: calmet@theory.caltech.edu

from a U(N) group. Furthermore it turns out that even in the U(1) case, charges are quantized [6,7] and it thus is impossible to describe quarks.

There is a framework that enables one to address these problems [8–11]. The aim of this new approach is to derive low energy effective actions for the non-commutative theory which is too complicated to handle. The matching of the non-commutative action to the low energy action on a commutative space-time is done in two steps. First the non-commutative coordinates are mapped to usual coordinates; the price to pay is the introduction of a star product. Secondly the non-commutative fields are mapped to commutative fields by means of the Seiberg–Witten maps. The Seiberg–Witten maps [12] are defined by the following requirement: ordinary gauge transformations $\delta A_\mu = \partial_\mu \Lambda + \mathrm{i}[\Lambda, A_\mu]$ and $\delta\Psi = \mathrm{i}\Lambda \cdot \Psi$ induce non-commutative gauge transformations of the fields $\hat{A}$, $\hat{\Psi}$: $\delta\hat{A}_\mu = \hat{\delta}\hat{A}_\mu$, $\delta\hat{\Psi} = \hat{\delta}\hat{\Psi}$.

The low energy action is local in the sense that there is no UV/IR mixing in that approach. The non-commutative nature of space-time is encoded in the higher order operators that enter the theory. The basic assumption is that the non-commutative fields are not Lie algebra valued but are in the enveloping algebra:

$$\hat{\Lambda} = \Lambda^0_a(x)T^a + \Lambda^1_{ab}(x) : T^aT^b : + \Lambda^2_{abc}(x) : T^aT^bT^c : + \dots \quad (3)$$

where : : denotes some appropriate ordering of the Lie algebra generators. One can choose, for example, a symmetrically ordered basis of the enveloping algebra, one then has $: T^a := T^a$ and $: T^aT^b := \frac{1}{2}\{T^a, T^b\}$ and so on. Taking fields in the enveloping of the algebra allows one to consider SU(N) groups. At first sight it seems that one has introduced an infinity number of degrees of freedom. It turns out that all fields appearing in (3) can be expressed in terms of the classical gauge parameter. Higher order terms in (3) are assumed to be suppressed by higher powers of θ.

Expanding to linear order in θ the star product and the non-commutative fields, one obtains the action [10]

$$\begin{aligned}
&\int \bar{\hat{\Psi}} \star (\mathrm{i}\gamma^\mu \hat{D}_\mu - m)\hat{\Psi} \mathrm{d}^4x \\
&= \int \bar{\psi}(\mathrm{i}\gamma^\mu D_\mu - m)\psi \mathrm{d}^4x \\
&\quad -\frac{1}{4}\int \theta^{\mu\nu}\bar{\psi}F_{\mu\nu}(\mathrm{i}\gamma^\alpha D_\alpha - m)\psi\mathrm{d}^4x \\
&\quad -\frac{1}{2}\int \theta^{\mu\nu}\bar{\psi}\gamma^\rho F_{\rho\mu}\mathrm{i}D_\nu\psi\mathrm{d}^4x - \frac{1}{4}\hat{F}_{\mu\nu} \star \hat{F}^{\mu\nu}\mathrm{d}^4x \\
&= -\frac{1}{4}\int F_{\mu\nu}F^{\mu\nu}\mathrm{d}^4x + \frac{1}{8}\int \theta^{\sigma\rho}F_{\sigma\rho}F_{\mu\nu}F^{\mu\nu}\mathrm{d}^4x \\
&\quad -\frac{1}{2}\int \theta^{\sigma\rho}F_{\mu\sigma}F_{\nu\rho}F^{\mu\nu}\mathrm{d}^4x. \qquad (4)
\end{aligned}$$

There are a number of difficulties which have to be addressed in order to formulate the standard model on a non-commutative space-time. These problems have been solved in [11].

The first problem is that one cannot introduce three different non-commutative gauge potentials. The reason is that non-commutative gauge invariance is linked to the invariance of the covariant coordinates $\hat{X}^\mu = \hat{x}^\mu + \hat{B}^\mu$. The Yang–Mills potential A_μ is related to B^μ by $B^\mu = \theta^{\mu\nu}A_\nu$, i.e. gauge transformations are related to transformations of the covariant coordinate. The solution is to introduce a master field: $V_\mu = g'A_\mu + gB_\mu + g_S G_\mu$ that contains all the gauge potential of the structure group SU(3)×SU(2)×U(1) and to performed a Seiberg–Witten map for $\hat{V}_\mu$. Note that a generalized gauge transformation is also introduced: $\Lambda = g'\alpha(x)Y + g\alpha_L(x) + g_s\alpha_s(x)$, with the Seiberg–Witten map $\hat{\Lambda} = \Lambda + \frac{1}{4}\theta^{\mu\nu}\{V_\nu, \partial_\mu\Lambda\} + \mathcal{O}(\theta^2)$.

The approach presented in [11] offers a very natural problem to the charge quantization problem. One introduces n different non-commutative hyperphotons, one for each charge entering the model: $\hat{\delta}\hat{a}^{(n)}_i = \partial_i\hat{\lambda}^{(n)} + \mathrm{i}[\hat{\lambda}^{(n)}, \hat{a}^{(n)}_i]$ with $\hat{\delta}\hat{\Psi}^{(n)} = \mathrm{i}eq^{(n)}\hat{\lambda}^{(n)} \star \hat{\Psi}^{(n)}$. At first sight, it seems that this implies the existence of n photons in nature, i.e. that the theory has too many degrees of freedom, but once again the Seiberg–Witten maps can be used to reduce the amount of degrees of freedom. It turns out that these n non-commutative hyperphotons have the same classical limit a_i: $\hat{a}^{(n)}_i = a_i - eq^{(n)}\frac{1}{4}\theta^{kl}\{a_k, \partial_l a_i + f_{li}\} + \mathcal{O}(\theta^2)$, i.e. there is only one classical photon.

Another problem are the Yukawa couplings: a non-commutative field can transform on the left-hand side or on the right-hand side and this makes a difference. This is an obvious complication for Yukawa couplings. For example $\bar{\hat{L}} \star \hat{\Phi} \star \hat{e}_R$ is not invariant under a non-commutative gauge transformation if $\hat{\Phi}$ transforms only on the right-hand side or only on the left-hand side. The solution [11] is to assume that it transforms on both sides to cancel the transformations of the SU(2) doublet and of the SU(2) singlet fields, $\bar{\hat{L}} \star \rho_L(\hat{\Phi}) \star \hat{e}_R$ with

$$\rho_L(\hat{\Phi}) = \Phi\left[\phi, -\frac{1}{2}g'\mathcal{A}_\mu + gB_\mu, g'\mathcal{A}_\mu\right]$$

and

$$\begin{aligned}
\widehat{\Phi}[\Phi, A, A'] &= \Phi + \frac{1}{2}\theta^{\mu\nu}A_\nu\left(\partial_\mu\Phi - \frac{\mathrm{i}}{2}(A_\mu\Phi - \Phi A'_\mu)\right) \\
&\quad + \frac{1}{2}\theta^{\mu\nu}\left(\partial_\mu\Phi - \frac{\mathrm{i}}{2}(A_\mu\Phi - \Phi A'_\mu)\right)A'_\nu.
\end{aligned}$$

It should be noted that the form of the operators that enter the effective theory is very severely constrained by the non-commutative gauge invariance. Naively one could guess that an operator $m\theta^{\mu\nu}\bar{\Psi}\sigma_{\mu\nu}\Psi$ could appear in the low energy effective action [13]. After all, the Wilsonian approach to effective theories teaches us that an operator not forbidden by a symmetry will enter the theory with potentially a coefficient of order one. But it is absolutely not clear that such an operator is compatible with the non-commutative gauge invariance and might thus be simply forbidden. One might argue that it is generated by a term $m\theta^{\mu\nu}\bar{\hat{\Psi}}\sigma_{\mu\nu} \star \hat{\Psi}$ that is invariant under non-commutative gauge transformations, but such an operator makes little sense since $\theta^{\mu\nu}$ only enters the theory through the star product and the Seiberg–Witten maps of the fields. One would have to show that such an operator can be generated at the loop level on the non-commutative side, which seems doubtful since the non-commutative action is non-perturbative in θ. One has to be very careful when effective theory arguments are applied to these models since it is very difficult to keep track of the fundamental symmetry which is the non-commutative gauge invariance.

Another source of model dependence originates in the choice of the definition of the trace in the enveloping algebra and of the representation of the non-commutative field strength $\widehat{F}^{\mu\nu}$. The action for non-Abelian non-commutative gauge bosons is

$$S_{\text{gauge}} = -\frac{1}{2}\int \mathrm{d}^4x\, \mathbf{Tr}\frac{1}{\boldsymbol{G}^2}\widehat{F}_{\mu\nu} \star \widehat{F}^{\mu\nu},$$

with the non-commutative field strength $\widehat{F}_{\mu\nu}$, an appropriate trace $\mathbf{Tr}$ and an operator $\boldsymbol{G}$. This operator must commute with all generators (Y, T_L^a, T_S^b) of the gauge group so that it does not spoil the trace property of $\mathbf{Tr}$.

The operator $\boldsymbol{G}$ is in general a function of Y and the Casimir operators of SU(2) and SU(3). However, due to the assignment of hypercharges in the standard model it is possible to express $\boldsymbol{G}$ using Y and six constants g_1, …, g_6 corresponding to the six multiplets. In the classical limit only certain combinations of these six constants, corresponding to the usual coupling constants g', g and g_S are relevant. The relation is given by the following equations: $1/g_1^2 + 1/(2g_2^2) + 4/(3g_3^2) + 1/(3g_4^2) + 1/(6g_5^2) + 1/(2g_6^2) = 1/(2g'^2)$, $1/g_2^2 + 3/g_5^2 + 1/g_6^2 = 1/g^2$ and $1/g_3^2 + 1/g_4^2 + 2/g_5^2 = 1/g_S^2$. The values of the traces, $\mathbf{Tr}\frac{1}{\boldsymbol{G}^2}Y^3$, $\mathbf{Tr}\frac{1}{\boldsymbol{G}^2}YT_L^aT_L^b$ and $\mathbf{Tr}\frac{1}{\boldsymbol{G}^2}YT_S^cT_S^d$, corresponding to triple gauge boson vertices are thus model dependent. One consequence is that the triple photon vertex cannot be used to bound space-time non-commutativity. While such an interaction can be seen as a smoking gun of space-time non-commutativity, the bounds obtained are model dependent and only constrain a combination of $\theta^{\mu\nu}$ and of an unknown coupling constant. It is worth noting that most collider studies have considered modifications of the gauge sector to search for space-time non-commutativity; see e.g. [14,15]. It has also been proposed to use rare decays based on modifications of the Seiberg–Witten map to search for space-time non-commutativity. While these channels and rare decays are interesting from the discovery point of view, they cannot be used to bound the non-commutative nature of space-time itself since the rate for these decays depend either on the choice for the trace in the enveloping algebra or on particular choices for the Seiberg–Witten maps.

The only model independent part of the effective action is the fermionic sector. There are two types of model independent bounds in the literature that are relevant to the case where fields are taken to be in the enveloping algebra.

The first relevant study is that of Carroll et al. [17]. They replace $F_{\mu\nu} \to f_{\mu\nu} + F_{\mu\nu}$ in (4), where $f_{\mu\nu}$ is understood to be a constant background field and $F_{\mu\nu}$ now denotes a small dynamical fluctuation. Keeping only terms up to quadratic order in the fluctuations and performing a physically irrelevant rescaling of the fields Ψ and A_μ to maintain a conventionally normalized kinetic term, they obtained $L = \frac{1}{2}\mathrm{i}\bar{\Psi}\gamma^\mu \overleftrightarrow{D}_\mu \Psi - m\bar{\Psi}\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\mathrm{i}c_{\mu\nu}\bar{\Psi}\gamma^\mu \overleftrightarrow{D}^\nu \Psi - \frac{1}{4}k_{F\alpha\beta\gamma\delta}F^{\alpha\beta}F^{\gamma\delta}$. They have replaced, in this equation, the charge q in the covariant derivative with a scaled effective value $q_{\text{eff}} = \left(1 + \frac{1}{4}qf^{\mu\nu}\theta_{\mu\nu}\right)q$. The coefficients $c_{\mu\nu}$ and $k_{F\alpha\beta\gamma\delta}$ are given by $c_{\mu\nu} = -\frac{1}{2}qf_\mu^\lambda\theta_{\lambda\nu}$ and $k_{F\alpha\beta\gamma\delta} = -qf_\alpha^\lambda\theta_{\lambda\gamma}\eta_{\beta\delta} + \frac{1}{2}qf_{\alpha\gamma}\theta_{\beta\delta} - \frac{1}{4}qf_{\alpha\beta}\theta_{\gamma\delta} - (\alpha \leftrightarrow \beta) - (\gamma \leftrightarrow \delta) + (\alpha\beta \leftrightarrow \gamma\delta)$. $k_{F\alpha\beta\gamma\delta}$ is only very weakly constrained by experiments. That constraint would be model dependent since these coefficient depends on the choice of the representation for the non-commutative gauge fields and thus on the way the trace in the enveloping algebra is defined. On the other hand the coefficient $c_{\mu\nu}$ is accessible through clock comparison studies and is directly related to the fermionic sector of the action. Carroll et al. obtain the bounds $|\theta^{YZ}|, |\theta^{ZX}| \le (10\,\text{TeV})^2$ using a rather crude model for the ^{9}Be nucleus wavefunction.

The other constraint on space-time non-commutativity relevant to the case where the non-commutative fields are taken to be in the enveloping algebra comes from a study by Carlson at al. [18]. They study non-commutative QCD at the one loop order. They considered the one loop correction to the quark mass and wavefunction renormalization and performed their calculation using the low energy effective action (4). The one loop expression needs to be regularized, the authors of [18] choose to do so by a Pauli–Villars regularization procedure. While they are very careful not to break the classical gauge invariance, there is a priori no guaranty that such a procedure respects the non-commutative gauge invariance. But let us assume that the Pauli–Villars regulator respects both symmetries. The result obtained in [18] is, keeping just the $O(\theta)$ terms,

$$\mathrm{i}\mathcal{M}(\lambda^2, M^2) = \frac{2}{3}g^2\{(\not{p} - m), \sigma_{\mu\alpha}\} \times \int \frac{(\mathrm{d}q)}{(q^2-\lambda^2)\left((p+q)^2 - M^2\right)} q^\alpha\theta^{\mu\nu}(p+q)_\nu\ , \quad (5)$$

where $\{(\not{p} - m), \sigma_{\mu\alpha}\} = (\not{p} - m)\sigma_{\mu\alpha} + \sigma_{\mu\alpha}(\not{p} - m)$. The Pauli–Villars regulated amplitude is then given by $\mathcal{M} \to \mathcal{M}(0, m^2) - \mathcal{M}(\Lambda^2, m^2) - \mathcal{M}(0, \Lambda^2) + \mathcal{M}(\Lambda^2, \Lambda^2)$, where Λ is a large mass scale. Their result is

$$\mathcal{M} = \frac{g^2}{96\pi^2}\left(\left\{(m - \not{p}),\ \Lambda^2\theta^{\mu\nu}\sigma_{\mu\nu}\right\} - \frac{2}{3}\left\{(m - \not{p}),\ p_\mu\theta^{\mu\nu}\sigma_{\nu\tau}p^\tau \ln \Lambda^2\right\}\right)\ , \quad (6)$$

for the term leading in Λ for each Dirac structure. The authors of [18] considered the three operators

$$m\theta^{\mu\nu}\bar{q}\sigma_{\mu\nu}q, \quad \theta^{\mu\nu}\bar{q}\sigma_{\mu\nu}\not{D}q, \quad \text{and} \quad \theta^{\mu\nu}D_\mu\bar{q}\,\sigma_{\nu\rho}D^\rho q, \quad (7)$$

and obtained, using the first of these operators, the bound

$$\theta\Lambda^2 \lesssim 10^{-29}\ , \quad (8)$$

where Λ is an ultraviolet regularization scale. But these operators enter the game in a very specific combination. A closer look at (6) reveals that the matrix element is vanishing. Since we are working just to first order in the operators (7) the QCD equations of motion $(\mathrm{i}\not{D}-m)q=0$ can be used [19]. This invalidates the bound (8) and is a very strong indication that these operators are forbidden by the non-commutative gauge invariance. It should be noted that there is another approach to loop calculations [23], where the non-commutative action is first regularized and then the effective theory is derived using the Seiberg–Witten maps. This approach is the most promising since it ensures that the result of loop calculations is gauge invariant at the non-commutative level too. This method has been applied to anomalies, but not yet to the calculation of observable quantities. It would be of great interest to verify if the result obtained in this paper, i.e. the vanishing of the one loop contribution to the quark mass and wavefunction renormalization, could be confirmed using the approach developed in [23].

We have shown that the bounds on the non-commutativity of space-time are fairly loose if fields are taken to be in the enveloping algebra, and are only of the order of 10 TeV. Much more effort has to be invested to derive bounds on the non-commutative nature of space-time. It is important to realize that any bound is framework dependent and even in a given framework there is, most of the time, some model dependence. We have a clear idea of what signal would have to be interpreted as evidence for the non-commutativity of space-time; on the other hand bounding the non-commutative parameter $\theta^{\mu\nu}$ is a very difficult task.

The fact that the bounds are of the order of 10 TeV should not be taken as an indication that colliders studies are useless. It is conceivable that $\theta^{\mu\nu}$ is not a constant but a more complicated function. As it has been argued in [4], the higher order operators that describe the non-commutative nature of space-time might very well be energy-momentum dependent and thus only become relevant at high energies or equivalently at short distance. This should be a very strong motivation to study more model independent contributions to particle reactions that can be studied at the next generation of colliders. Some work in that direction [20–22] has already been done, but much more remains to be done.

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